Analysis of the Wells Report

Summary

The conclusions of the Wells Report[1] ultimately depend on statistical and technical analysis carried out by Exponent, their technical consultants. The original problem, as framed by Exponent, was whether the observed pressure drop of Patriot balls could be explained by physical or environmental factors, including temperature changes and selection of pregame gauges:

> We then sought to determine whether any combination of the factors listed in 7a through 7d [temperatures at pre-game, on the field and at half-time; timing of half-time measurements; wetness; pre-game gauge use] above (within ranges defined as realistic by Paul, Weiss) suggested pressure levels that matched those recorded on Game Day. If those factors could be set in such a way that the pressures suggested by the transient experiments matched the Game Day measurements, then we could conclude that the Game Day measurements could be explained by physical or environmental factors.

Exponent studied a number of permutations of factors, claiming that none of these combinations accounted for the additional loss of air pressure in Patriot balls or the difference in pressure loss in respect to Colt balls:

> Exponent concluded that, within the range of likely game conditions and circumstances studied, they could identify no set of credible environmental or physical factors that completely accounts for the Patriots halftime measurements or for the additional loss in air pressure exhibited by the Patriots game balls, as compared to the loss in air pressure exhibited by the Colts game balls. Dr. Marlow agreed with this and all of Exponent's conclusions. This absence of a credible scientific explanation for the Patriots halftime measurements tends to support a finding that human intervention may account for the additional loss of pressure exhibited by the Patriots balls.

In this article, I show that these factors can, in fact, be set "in such a way that the pressures suggested by the transient experiments matched the Game Day measurements" as follows:

- Pre-game temperature around 71 deg F
- Logo measurement of Patriot balls and Non-Logo measurement of Colt balls

It is therefore possible to unequivocally say that the "Game Day measurements could be explained by physical or environmental factors", contradicting the key technical finding of the Wells Report. The corollary is that the Wells Report provides no technical basis for concluding that the Patriot balls had even been out of compliance with NFL regulations during the AFC Championship.

In previous discussions of the Wells Report, Prof MacKinnon[2] and Hassett et al[3] previously identified the important possibility that referee Anderson had not used the same gauge for pre-

[1] In February 2015, the NFL retained Ted Wells' firm (Paul, Weiss) to investigate the incident. Wells, in turn, engaged a firm of technical consultants (Exponent). Wells reported (https://www.aei.org/wp-content/uploads/2015/06/On-the-Wells-report.pdf) on May 4, 2015, including Exponent's technical report as an attachment to the Wells Report.

[2] Roderick MacKinnon, May 2015. Professor MacKinnon's Scientific Conclusion. http://wellsreportcontext.com/mackinnons-scientific-conclusion/

[3] Kevin A. Hassett, Joseph W. Sullivan, and Stan A. Veuger, June 2015. On the Wells Report.

game measurements of both teams – an inconsistency that also occurred in the half-time measurements under the supervision of NFL Executive Vice President Vincent. The present article extends their work to include analysis of Exponent's simulations and transients, showing that all relevant issues raised in the Wells Report can be fully explained by "physical and environmental factors".

The Wells Report also revealed remarkable chaos and inefficiency in NFL protocols and procedures, even in connection with half-time measurements under the additional scrutiny of NFL Executive Vice President Vincent and other senior NFL officials. Had their protocols met reasonable standards, much, if not most, of the present, seemingly false, controversy could have been avoided.

Introduction

The conclusions of the Wells Report[4] ultimately depend on statistical and technical analysis carried out by Exponent, their technical consultants. The overall chronology of events, together with expected pressure changes, is summarized in the figure from the Wells Report shown below.

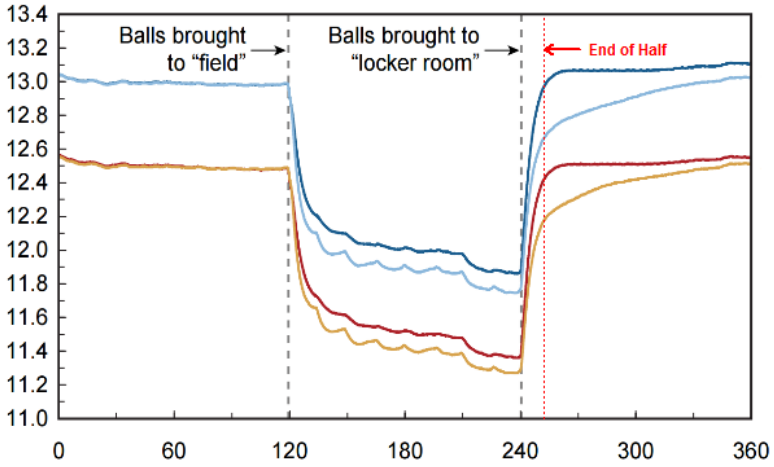

Figure 1. Figure 21 from Wells Report showing pro forma pressures for Colt (blue) and Patriot (red) footballs, initialized at 13 psig and 12.5 psig respectively under changing temperature conditions as described in text. Both dry (higher) and wet balls (lower) are shown. Overplotted is a vertical red line showing the end of the half-time intermission.

Balls were measured pre-game in the officials' locker room at a temperature estimated to be between 67-71 deg F. Patriot balls were set at 12.5 psig, while Colt balls were observed to be 13.0-13.1 psig. The ambient temperature outside was 48 deg F by the end of half-time and it was heavily raining.

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As the temperature of the initially warm balls decreased during the first half, pressures also declined according to the Ideal Gas Law. The Wells Report estimated that pressure of dry footballs would decrease to 11.32-11.5 psig (based on a range of 67-71 deg F in the pre-game officials' room). The half-time pressure is sensitive to pre-game temperature: at 71 deg F initialization, the expected half-time pressure would be 11.32 psig, but would be 11.5 psig at 67 deg F initialization - about 0.1 psi lower per 2 deg F higher pre-game temperature. The Wells Report also observed that pressures of wet footballs could up to 0.3-0.45 psi lower[5], though it did not offer any physical explanation of this phenomenon. These values were in line with earlier blog and news commentary in which physicists were more than somewhat surprised at the sudden interest in the Ideal Gas Law[6].

After the first half, the balls were returned to an officials' room for 13.5 minutes at a temperature of 73-74 deg F and (dry) relative humidity of ~20%, where their temperatures and pressures were expected to rise rapidly.

Such warming is governed in the first instance by Newton's Law of Cooling (Warming), though this is not specifically mentioned in the Wells Report, with pressures determined in turn from the Ideal Gas Law at the transient ball temperature. The combination of wet footballs and low relative humidity in the officials' room appears to be a recipe for evaporative cooling[7], an important phenomenon which could easily be relevant at the small discrepancies in question, but which was not discussed or formally excluded in the Wells Report.

Exponent's Reasoning

Exponent had attempted to address the issue of whether the observed pressure drop of Patriot balls could be explained by physical or environmental factors, including temperature changes and selection of pregame gauges:

> We then sought to determine whether any combination of the factors listed in 7a through 7d [temperatures at pre-game, on the field and at half-time; timing of half-time measurements; wetness; pre-game gauge use] above (within ranges defined as realistic by Paul, Weiss) suggested pressure levels that matched those recorded on Game Day. If those factors could be set in such a way that the pressures suggested by the transient experiments matched the Game Day measurements, then we could conclude that the Game Day measurements could be explained by physical or environmental factors.

The issue of pre-game gauges arises because referee Anderson had two gauges, one of which (the "Logo Gauge") measured ~0.38 psi higher than the other. The observed average Patriot pressure using the Logo Gauge was 11.49 psig, apparently well within the expected range of 11.32-11.5 psig. Many commentators, including MacKinnon and Hassett et al, have argued that

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[5] The value of 0.3 psi is referred to in the text, but the difference between wet and dry differentials in Figures 26 and 27 is ~0.45 psi.

[6] Pathfinder: https://www.youtube.com/watch?v=_Gd0kGhIcF4; Headsmart Labs https://www.youtube.com/watch?v=CxsXFX3tDpg

[7] There are numerous online demonstrations of evaporative cooling, e.g. https://www.youtube.com/watch?v=dt8KFgqs2A4; https://www.youtube.com/watch?v=nyczz9NsFgs; https://www.youtube.com/watch?v=cwyDsHG-ymQ. Basketball leather in older NBA balls was been shown to quickly absorb 70 g of moisture (http://blogmaverick.com/2006/10/27/nba-balls/).

use of the Logo Gauge for pre-game measurement of Patriot balls in itself appears to fully explain Patriot Game Day measurements under elementary physics.

Exponent was aware of this line of reasoning and concluded otherwise[8], using four main lines of argument, each of which has major flaws and which are not entirely consistent.

Their first main argument was the supposedly "statistically significant" difference in observed pressure drop between Patriot and Colt balls, summarized by Wells as follows:

> According to both Exponent and Dr. Marlow, the difference in the average pressure drops between the Patriots and Colts footballs is statistically significant. This conclusion was consistent regardless of the assumptions made as to which of the two gauges was used to measure the game balls prior to the game and at halftime.

The second main argument came from Exponent's simulations of "Game Day" conditions, simulations in which Exponent attempted to replicate a mix of wet and dry footballs coming from outside temperatures to the officials' room and then being measured in a plausible sequence. Exponent reported that they replicated Colt measurements, but failed to explain the observed Patriot measurements, even under Logo Gauge initialization:

> They also produced average pressure measurements for the Colts footballs similar to those recorded for the Colts at halftime on Game Day. However, the experimental simulations failed to explain the measurements recorded for the Patriots on Game Day... The averages of the Patriots measurements recorded during each of the experimental simulations using the Logo Gauge to set the footballs were also generally higher than the average of the Patriots Game Day halftime measurements, and the only way to achieve measurements for the Patriots balls similar to those recorded on Game Day in experimental simulations using the Logo Gauge was to start the simulated halftime testing immediately once the footballs arrived in the simulated Locker Room, which is earlier than we are told is realistic. In fact, the average Patriots measurements from Game Day are lower than the lowest average attained by the Exponent simulations.

In a closely related argument, Exponent estimated pressure transients during the half-time intermission for dry and wet balls and compared these transients to Patriot and Colt observations. Even with Logo Gauge initialization, they concluded that there was no plausible intersection. They found no overlap between the dry transient and the observed average and no overlap with the transient and the observed average after two minutes. They found it unlikely that Patriot measurements could have been made quickly enough to meet the two minute window for the average or that they could average out to the wet transient, summarized by Exponent as follows:

> if the Logo Gauge was used pre-game, the Patriots average halftime measurement will match the pressures predicted by the transient curves (with the Colts halftime measurements also matching the predicted range), but only if the testing of the Patriots balls began immediately once the footballs arrived in the Officials Locker Room at halftime and took no more than 4 minutes, and only if the majority of the Patriots game balls were wet. As noted, testing of the Patriots balls is likely to have begun no sooner than 2 minutes and is likely to have taken approximately 4 to 5 minutes. Further, based on statements made to Paul, Weiss (and

[8] Exponent concluded that, within the range of likely game conditions and circumstances studied, they could identify no set of credible environmental or physical factors that completely accounts for the Patriots halftime measurements or for the additional loss in air pressure exhibited by the Patriots game balls, as compared to the loss in air pressure exhibited by the Colts game balls. Dr. Marlow agreed with this and all of Exponent's conclusions. This absence of a credible scientific explanation for the Patriots halftime measurements tends to support a finding that human intervention may account for the additional loss of pressure exhibited by the Patriots balls.

> subsequently conveyed to Exponent) by Patriots ballboys and game officials, we understand that some of the Patriots game balls may have been damp when tested at halftime, but none were waterlogged.

Their other main line of argument was that the Logo Gauge was not used for pre-game measurements. Their argument on this point is expressed in several locations, some of which make no sense – a point that has not been lost on blog commenters. In the most coherent version of this argument, Exponent observed that referee Anderson's pre-game measurements were more or less consistent with pressures as tendered by the two teams; that the Non-Logo Gauge was relatively consistent with a properly calibrated Master Gauge, whereas the Logo Gauge was biased high by ~0.28 psi; that it was "very unlikely" that both teams would have gauges with similar (slight) high biases; and, therefore, that it was "most likely" that Anderson used the Non-Logo Gauge.

> Walt Anderson recalled that according to the gauge he used (which is either the Logo or Non-Logo Gauge), all of the Patriots and Colts footballs measured at or near 12.5 psig and 13.0 psig, respectively, when he first tested them (with two Patriots balls slightly below 12.5 psig). This means that the gauges used by the Patriots and the Colts each read similarly to the gauge used by Walt Anderson during his pregame inspection.
>
> This means that the gauges used by the Patriots and the Colts each read similarly to the gauge used by Walt Anderson during his pregame inspection. It has been shown that the Logo Gauge consistently reads higher than all other gauges analyzed in this investigation. As a result, it is very unlikely that the Logo Gauge would have read similarly to the gauges used by each team. Therefore, it is most likely that the gauge used by Walt Anderson prior to the game was the Non-Logo Gauge, which read similarly to the Master Gauge and other gauges tested during the investigation. (E, p 44).

Challenges to the Wells Report

Both MacKinnon and Hassett et al convincingly show that Exponent failed to address a plausible pre-game gauge scenario. Exponent had implicitly assumed that referee Anderson used the same gauge for pre-game measurement of both Patriot and Colt balls. However, there is no justification for this assumption, as evidenced by events at half-time, where, according to Exponent, it appears that NFL officials Prioleau and Blakeman had inattentively exchanged gauges between measuring Patriot and Colt balls.

MacKinnon and Hassett et al both point out that identical inattentiveness to gauge selection by referee Anderson in his pre-game measurements would fully account for the pressure drop differential. MacKinnon summarized the scenario as follows (Hassett et al propose the identical scenario):

> Is it possible that the same Official could use one gauge for the Patriots and the other for the Colts measurements? Not only is this possible but it is exactly what happened at halftime. The Wells Report describes a detailed procedure in which each Official used one gauge to measure pressures of 11 Patriots balls first then 4 Colts balls. Only on subsequent data analysis did it become evident that the gauges were inadvertently switched in between measuring the team balls. It is very easy to understand how this could happen because the gauges look almost identical. This could also have occurred for the pregame measurements because the Official who made those measurements owned both gauges and brought them to the stadium. Imagine the Official has a bunch of balls from each of two teams that he has to measure and two gauges that are almost identical, so much so that they were interchanged during the rigid protocol of recording described for halftime…
>
> Finally, the claim of a statistically significant difference in pressure drop between the two team balls regardless of which gauge was used did not account for the fact that the Colts balls were apparently measured

at the end of halftime since the officials ran out of time and made only four measurements – in other words, the Colts balls were measured after the Patriots balls and had warmed up more.

The pre-game gauge usage proposed by MacKinnon and Hassett et al corresponds to the gauges used by alternative referee Prioleau at half-time (according to the Wells Report interpretation). In the rest of this article, I'll denote this setup as the "Prioleau" Gauge assumption.

Outstanding Issues

While both MacKinnon and Hassett et al show that the difference in pressure drops can be explained by physical or environmental factors using the "Prioleau" Gauge assumption, neither addressed Exponent's simulations and transients or the Non-Logo Gauge, but it turns out that the Prioleau Gauge sequence also has a major impact on these analyses as well and that all issues and discrepancies raised in the Wells Report can be fully resolved.

Half-Time and the "Prioleau" Gauge Sequence

At the end of the first half, Patriot and Colt footballs were collected and taken to the officials' room, where pressures were measured by alternate NFL officials Clete Blakeman and Dyrol Prioleau, each using one of Anderson's gauges. The measurements were observed and supervised by three NFL officials: Troy Vincent, Executive VP of Football Operations; Dan Grossi, Director of Event Security; and Richard Farley, NFL Security Representative for the New England Patriots. The measurements were written down by Farley, as they were called out.

None of the officials recorded which gauge was used by which referee, documenting only which official made which measurement (see Wells Report Table 2). For each pair of measurements, one measurement was always ~0.38 psi higher than the other. However, for all Patriot balls, Prioleau's measurement was about 0.35-.4 psi higher than Blakeman's, while, for three of four Colt balls, Blakeman's measurements were 0.3-.4 psi higher than Prioleau's. (The other followed the first pattern).

Exponent subsequently determined that the Logo Gauge was biased about ~0.38 psi high relative to the Non-Logo Gauge and reasonably concluded that the higher of each pair of measurements had been taken with the Logo Gauge. The Wells Report concluded that the officials had inadvertently exchanged gauges between measurement of Patriot and Colt balls[9]: i.e. Prioleau had used the Logo Gauge to measure Patriot balls and the Non-Logo Gauge to measure the Colt balls. Rather than assuming two further swaps (before the third Colt ball and again before the fourth Colt ball), they concluded that Farley had inadvertently reversed the readings of the third Colt ball.[10]

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[9] "it appears most likely that the two officials switched gauges in between measuring each team's footballs, meaning that Blakeman most likely used the Logo Gauge and Prioleau most likely used the Non-Logo Gauge to test the Colts balls at halftime". WR, fn41 p69

[10] "For the reasons described in Section VII.B, we believe it is more probable that Anderson used the Non-Logo Gauge for his pre-game measurements n30 p 52; As noted above, we also believe that Walt Anderson most likely used the Non-Logo Gauge prior to the game n37 p 67; However, for a given set of measurements, the differential between the gauges generally remained consistent when compared to a calibrated gauge … Exponent relied upon this information, as well as the fact that during the testing the Non-Logo Gauge never produced a reading higher than the Logo Gauge, to conclude that Walt Anderson most likely used the Non-Logo Gauge to inspect the game balls prior to the game." p116.

Anderson could easily have used gauges as Prioleau is presumed to have done. Suppose that Anderson measured Colt balls using the Non-Logo Gauge and then put his gauge back in his pocket. His selection of gauge for measuring Patriot draws would then be a random draw from his pocket, equally likely to choose the Logo Gauge as the Non-Logo Gauge.

Reconciling the Pressure Drop Differential

The Patriot-Colt pressure drop differential from pre-game pressures to Prioleau's half-time measurements (equivalent to Anderson using the Logo Gauge for Patriots and Non-Logo Gauge for Colts) was only 0.29 psi, the difference between the Patriot pressure drop of 1.01 psi (Logo) and Colt pressure drop of 0.72 psi (Non-Logo), as shown below:

	Patriot (Logo)	**Colt (Non-Logo)**
Pre-Game[11]	12.5	13.05
Half-time assuming Patriot Logo and Colt Non-Logo[12]	11.49	12.33
Difference	1.01	0.72
Pressure Difference		**0.29**

Both MacKinnon and Hassett et al correctly observed that this reduced difference is readily explained through nothing more than later measurement in the intermission of Colt balls than Patriot balls. This can be confirmed quantitatively by the Wells Report's own simulations (See their Tables 13 and 14). The average difference between Patriot pressure drop and Colt pressure drop in the Wells Report simulations was 0.30 psi.

All of Exponent's simulations assumed that Colt measurements were carried out prior to Patriot reflation. Surprisingly, Exponent stated that there was "some uncertainty" about whether this assumption was correct, an astounding uncertainty under the circumstances. If Colt measurements were carried out towards the end of the intermission, after Patriot reflation, then there would be an even larger window.[13]

[11] The Wells Report considers alternatives in which Colt balls were initialized to 13 and 13.1 psig; 13.05 psig is therefore used.

[12] After correcting the apparent transposition of the third Colt reading, as recommended and applied by the Wells Report.

[13] Both MacKinnon and Hassett et al observe this possibility. As circumstantial support, only four Colt balls were measured in the intermission. The Wells Report purported to justify this on the basis that time was "running out" before the end of the intermission, but, seemingly inconsistently, also presumed for the purposes of simulations and transients that the Colt balls were measured in mid-intermission.

The Simulations and Wells Report Figure 30

As noted in the introduction, one of the key lines of argument of the Wells Report was Exponent's analysis of Game Day simulations using the Logo Gauge, an issue not addressed by either MacKinnon or Hassett et al.

Exponent had attempted to simulate Game Day conditions and measurements using a mix of dry and wet balls. Their simulations based on Logo Gauge initialization (Patriot –solid red dots; Colts – solid steelblue dots) were shown in their Figure 30, which showed results under varied average measurement time: average Patriot measurement times were in the 2-6 minute range and average Colt measurement times were in the 7-9 minute range. Their Figure 30 also shows an envelope estimated transients for wet and dry balls (Patriot – reddish; Colt – blue) and average observations (horizontal lines). I have annotated and overplotted this figure as described below.

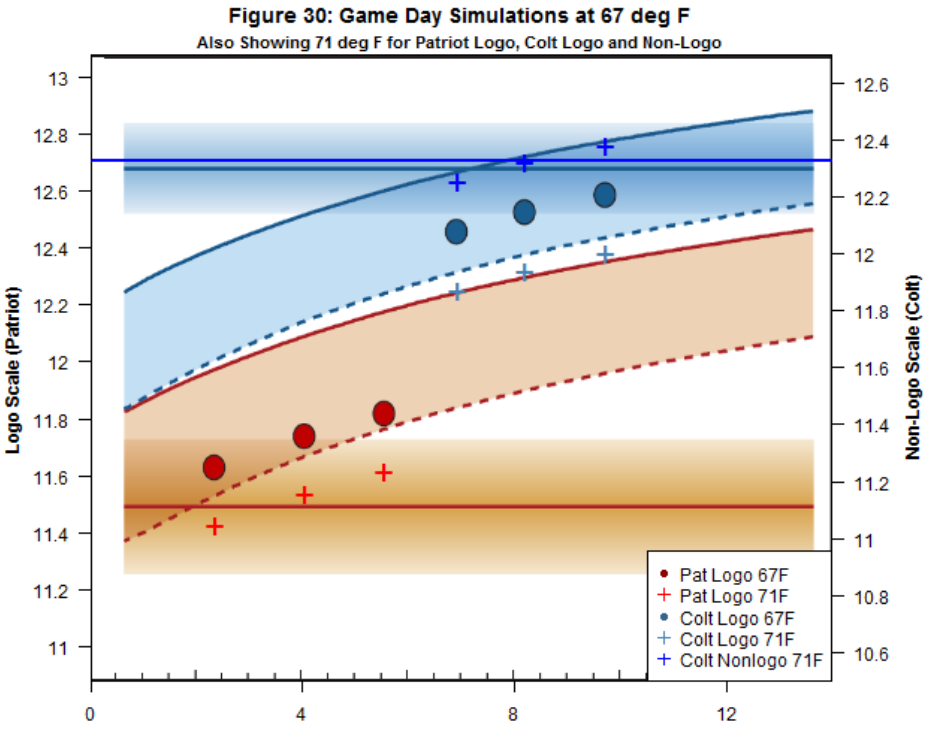

Figure 2. Annotated version of Wells Report Figure 30. To simplify comparison with the observations taken with Logo and Non-Logo Gauges, the scales on the left and right axis are Logo and Non-Logo scale, rather than the "correct" Master Gauge scale of the original diagram. Horizontal blue line shows actual average of 4 Colt measurements. In each case, the scales have been centered using the Patriot half-time average as a reference: 11.49 psig (Logo) and 11.1 (Non-Logo).[14] *Plus signs: Patriot 71 deg F (red); Colt 71 deg F (Logo-steelblue; Non-Logo blue).*

Although Wells claimed that Exponent had considered "all permutations", the simulations and transients of Figure 30 (and 27) were carried out at 67 deg F – the temperature most adverse to the Patriots – rather than 71 deg F. (Exponent purported to justify this adverse assumption, but their justification does not stand up, particularly to the gauge assumption, as discussed later.)

[14] Figure online http://www.climateaudit.info/data/football/ figure30_with_71_deg_simulations.png

Simulations at 71 deg F initialization will be ~0.2 psi lower than at 67 deg F initialization under the Ideal Gas Law. These corresponding results at 71 deg F initialization are overplotted in Figure 2 with + signs (Patriot Logo – red; Colt Logo – steelblue).

For Patriot simulations between 1.5 and 4 minute average measurement time, there is now negligible difference between the simulation results and the observed Patriot average. The earliest simulation is now below the Patriot average, contradicting one of the major technical conclusions of the Wells Report – that "the average Patriots measurements from Game Day are lower than the lowest average attained by the Exponent simulations".

Even at 67 deg F initialization, the difference between simulated values and the observed Patriot average is only 0.12-0.24 psi for the simulations with average Patriot measurement times under 4 minutes - amounts that are an order of magnitude less than the impacts from the Ideal Gas Law (1.2-1.65 psi depending on wetness). The differentials are also within the uncertainty envelope shown in Figure 30. In my opinion, these exculpatory results (at both temperature) should have been clearly reported by Exponent and such clear report carried forward to their summary. Instead, Exponent only stated that the Patriot simulation results were "noticeably higher" and "all higher" than the observed average:

> In …the Logo Gauge simulations, we see that the average measurements for the Colts footballs are generally at or near the line representing the average measurements from Game Day. In contrast, **all of the average measurements for the Patriots footballs generated by the simulations are noticeably higher than the line representing the average measurements from Game Day…**
>
> the Colts measurements [simulations] are all relatively close to the Game Day measurements, whereas the Patriots averages [simulations] are **all higher** than those calculated from the Game Day data…

The above diagram shows that all that is required to remove the very slight difference between simulations and observed average is to carry out the simulations at 71 deg F – a plausible temperature within the estimated pre-game range and, indeed, the temperature used in the Non-Logo simulations, but not reported in the Wells Report for the Logo simulations.

The 67 deg F Restriction

Exponent's stated rationalization for restricting simulations to 67 deg F was stated as follows (in connection with Figure 27):

> However, the pre-game temperature was set at 67°F because this was the only temperature that allowed the Colts balls to subsequently reach their average pressure during the simulated Locker Room Period. Any pre-game temperature that was higher than 67°F resulted in the Colts balls reaching the Game Day halftime average pressure later than 13.5 minutes into the Locker Room Period.

This assertion is incorrect and can easily be seen to be incorrect by merely inspecting Wells Report Figure 30 (Figure 2 above), which shows the Colt dry transient at 67 deg F crossing the steelblue horizontal line (observed Colt mean) at approximately 7.1 minutes. Transients at higher initialization temperatures will cross to the right of this intersection, but there is plenty of room for higher temperatures, including 71 deg F. The contradiction is so manifest that one has to presume that Exponent must have had a more substantive rationale.

One possible rationale – which also fails against gauge assumptions, but less grotesquely – can be interpreted from the Colt simulations in Figure 2 above. At 67 deg F Logo initialization, there is a noticeable discrepancy between Colt simulations and observed averages, with Colt simulations being about as far below the observed line as the Patriot simulations were above. Even though the discrepancies are comparable in magnitude, Exponent described the discrepancies in very different language,[15] with the slight Patriot difference being described as "noticeable", but not the similar Colt difference. The discrepancy for Colt simulations at 71 deg F initialization (steelblue + signs) is larger than at 67 deg F – about 0.38 psi at 71 deg F. Exponent's rationale for 67 deg F may be based on the opinion that the Colt discrepancy at 67 deg F initialization was the maximum tolerable.

The similarity of the Colt discrepancy at 71 deg F to the known bias between gauges is the sort of thing that ought to give pause to any data analyst. If Anderson had used the Non-Logo Gauge for Colt initialization (the "Prioleau" gauge assumption), the location of the simulations would be translated upward by the bias between gauges (~0.38 psi), as shown by the blue + signs in Figure 2. These almost exactly coincide with the observed average, shown here in a solid blue line. (The steelblue line of Wells Report Figure 30 is not quite on the observed average.)

Rather than Exponent's Logo simulations demonstrating under-inflation that cannot be explained by the Ideal Gas Law, they offer convincing additional support for the "Prioleau" gauge assumption.

The Transients

The third important leg of Exponent's argument was their argument that it was implausible that measurement of Patriot footballs took place early enough in the intermission to intersect the modeled wet and dry transients. They presented this argument in their Figure 27, the components of which were carried forward into Figure 30. They observed that there was no intersection between the horizontal line and the dry transient and that intersection with the wet transient was at ~2 minutes in the intermission (see Figure 3 below adapting their Figure 27).

Once again, their argument was based on transients calculated at the temperature most unfavorable to Patriots: 67 deg F initialization, an assumption that cannot be maintained as shown above. Transients at 71 deg F initialization are ~0.2 psi lower and thus to the right, creating a later window.

Secondly, it appears to me that there is an actual error in the Wells Report transients, which are implausibly high even at 67 deg F, thereby accentuating the supposed case against the Patriots.

In Figure 3 below, I've shown the original scale of Figure 27 (Master Gauge scale) on the right axis and, for more convenient reference, on the left scale, shown a Logo scale by centering with respect to the Patriot observed average (11.49 psig). I've discussed scale conversion issues in Appendix 1. From the Ideal Gas Law (see Wells Report Table 10), the half-time pressure for Colt balls at 67 deg F initialization should be 12.0 psi, and for Patriot balls about 11.5 psi.

[15] If Colt simulations were characterized as being "relatively close" or "generally at or near" observations, then the same language should have been used to describe Patriot simulations.

However, the dry transients of Figure 30 (and 27) intercept the y-axis about 0.2 psi too high. This effect is additional to the ~0.2 psi from using 67 deg F initialization, rather than 71 deg F initialization.

From Newton's Law of Cooling/Warming (see Appendix 2), the functional form of warming during half-time – exclusive of evaporative cooling - is a negative exponential to an asympotote, yielding the same functional form for pressure gain during half-time:

$$P(t) = P_{asym} - (P_{asym} - P_0) * \exp(-C * t)$$

where P_{asym} is the equilibrium pressure at the temperature of the officials' room (73-74 deg F), P_0 is the pressure at the end of the first half and the "restitution coefficient" C is a property of the footballs. The average value of the coefficient in fits to digitization of Wells Report Figures 25 and 27 transients is 0.115 (sd - 0.0066), a value used below.

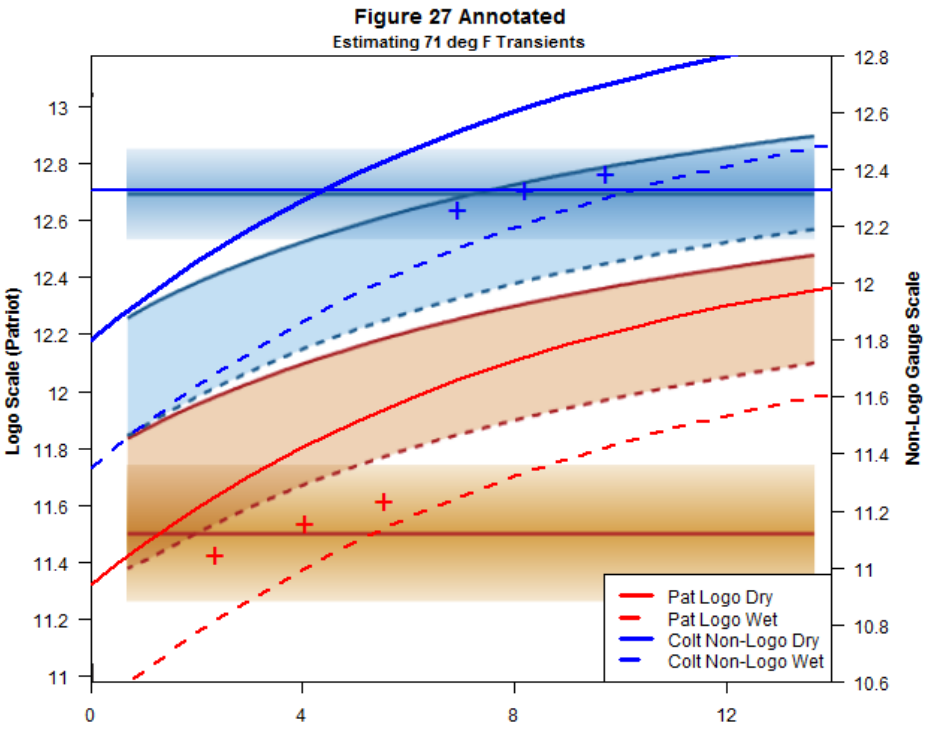

Figure 3. Annotation of Wells Report Figure 27 as in Figure 3 above. Overplotted are dry (solid) and wet (dashed) Patriot and Colt transients, re-estimated at 71 deg F with Colt transients based on Non-Logo Gauge initialization[16]. The scale of the original figure (Master Gauge) is shown on right axis; left axis shows Logo scale, centered on Patriot mean (11.49 psig). Carried forward for reference are simulation values (+ signs) from Figure 2.

Everything ties together. Both Colt and Patriot simulations (71 deg F initialization) are well within the transients. The window for average Patriot measurement time is from 1.3 to 5.2

[16] Figure online at http://www.climateaudit.info/data/football/figure27_with_71_deg_transients.png

minutes, no longer implausibly early but easily consistent with information on measurement of Patriot balls.[17]

Exponent's Argument for the Non-Logo Gauge

Exponent's fourth main argument was that it was "very likely" that Anderson had used the Non-Logo Gauge for pre-game measurements. If so, the discrepancies between Patriot simulations and observed average (negligible for the Logo Gauge) would be ~0.38 psi - the bias between gauges.

In my opinion, Exponent's argument purporting to support pre-game use of the Non-Logo Gauge is so defective that it is hard to imagine it being presented by licenced engineers. The absurdity of Exponent's argument has attracted some derisive commentary[18], but the issue was not forcefully addressed either by MacKinnon or Hassett et al.

The clearest exposition of Exponent's reasoning (see extended quote below) that team gauges had yielded measurements more or less similar to Anderson's pre-game measurements and that it was "very unlikely" that "each team" could have used a gauge with similar bias to Anderson's Logo Gauge. (They did not consider the possibility that the issue might merely be whether the Patriot gauge had similar bias to Anderson's Logo Gauge.)

> According to information provided by Paul, Weiss, personnel from both the Patriots and the Colts recall gauging the footballs for their teams to pressures at or near 12.5 psig and 13.0 psig, respectively, prior to providing the balls to Walt Anderson. Each team used its own gauge to adjust the final pressures before presenting the balls to the referee, who used a gauge different from either used by the two teams to measure the pressure in the footballs. Walt Anderson recalled that according to the gauge he used (which is either the Logo or Non-Logo Gauge), all of the Patriots and Colts footballs measured at or near 12.5 psig and 13.0 psig, respectively, when he first tested them (with two Patriots balls slightly below 12.5 psig). **This means that the gauges used by the Patriots and the Colts each read similarly to the gauge used by Walt Anderson during his pregame inspection.**

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[17] For the Patriots, it appears that so long as the average time at which the Patriots balls were measured is no later than approximately 2 minutes after the balls were brought back into the Officials Locker room, the Game Day results can be explained by natural causes. However, as noted above, the average measurement time for the Patriots footballs is unlikely to have been 2 minutes or earlier because testing of the Patriots balls is unlikely to have begun prior to 2 minutes into the Locker Room Period and was estimated to have taken approximately 4 to 5 minutes, according to information provided by Paul, Weiss (leading to an ending time of between 6 and 7 minutes and an average measurement time of between 4 and 4.5 minutes, assuming a start time of 2 minutes). Given the most likely timing of the measurements on Game Day, one would expect the average halftime pressure measured for the Patriots footballs on Game Day to be higher than what was actually recorded. In addition, for the Patriots halftime measurements to overlap the results predicted by the transient curves generated with the Logo Gauge, the majority of the Patriots footballs would have had to be wet. According to information collected during witness interviews conducted by Paul, Weiss, the Patriots ballboys attempted to keep the balls as dry as possible during the first half, and the game officials did not consider the balls to be overly wet when tested at halftime.

[18] Blogger Dave Garofolo (http://emailwagon.blogspot.ca/2015/05/how-i-trashed-wells-report-from-swan.html) ridiculed Exponent's reasoning on this issue as follows: "Let's say you drive a Nissan Maxima, and you want to know whether your car is faster than mine, but you have no idea what kind of car I drive. So you go out and buy several dozen Nissan Maximas and test them against yours. Surprise, surprise—the Maximas all perform at about the same speed! Based on these findings, you conclude that because your Maxima is so similar to all your test cars, it must also be similar to mine—even though you still don't have any clue what kind of car I drive. Oh, and you also conclude that since your other car, a Toyota Celica, is faster than the Maxima, then it must also be faster than mine. Everybody okay with that?" Many similar analogies can be conceived.

> This means that the gauges used by the Patriots and the Colts each read similarly to the gauge used by Walt Anderson during his pregame inspection. **It has been shown that the Logo Gauge consistently reads higher than all other gauges analyzed in this investigation. As a result**, **it is very unlikely that the Logo Gauge would have read similarly to the gauges used by each team.** Therefore, it is **most likely** that the gauge used by Walt Anderson prior to the game was the Non-Logo Gauge, which read similarly to the Master Gauge and other gauges tested during the investigation. (E, p 44).

Actual Colt and Patriot Gauges

An obvious question is why Exponent was guessing as to the readings of actual Colt and Patriot gauges, rather than analyzing them directly. According to the Wells Report, NFL officials were in possession of the Patriot gauge at half-time[19] and used it to test the pressure of an intercepted football. The Wells Report stated:

> We believe that [NFL official] Daniel located and used the pressure gauge supplied by the Patriots. We further believe that this is the gauge that John Jastremski considers his normal gauge.

However, the NFL was unable to produce this gauge when Exponent requested it:

> It [the gauge] has not been located since the day of the AFC Championship Game.

The NFL retained possession of the intercepted ball itself and, after the game, retained possession of all game balls. What happened to the gauge that Daniel used? Was it left behind by NFL officials in the officials' locker room? Did they return it to Jastremski? The Wells Report sheds no light on the matter. Nor did Exponent report calibration results from the Colt gauge used on game day and, indeed, was silent on whether they had made any efforts to "locate" the Colt game day gauge.

Exponent's Bizarre Experiment

As a supposed alternative to examining the actual Colt and Patriot gauges (the models of which are not even reported), Exponent purported to deduce their properties by studying the calibration of 49 (presumably) new Model CJ-01 (Non-Logo) gauges[20]. Exponent stated that they also attempted to obtain exemplar Logo Gauges, but were unsuccessful[21]. Their inability to obtain exemplar Logo Gauges indicates that referee Anderson might have had his Logo Gauge for a considerable period of time, opening up the possibility of drift – a phenomenon that Exponent reported in their own testing, though they did not address the potential impact on their gauge argument.

Exponent analyzed the readings of all 49 Model CJ-01 gauges at uniform 13.0 psig The average reading of these 49 (presumably new) gauges was 12.94 psig relative to a correct reading of 13.0 psig, a slight negative bias of 0.06 psi. The range of readings was 0.65 psi (from 12.55 to 12.3 psig).

Relative to a Master Gauge correctly calibrated, the Logo Gauge read ~0.38 psi too high (see Appendix 1 for discussion of somewhat inconsistent information), higher than all the new Non-

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[19] It has not been located since the day of the AFC Championship Game. (WR, Fn 36)

[20] A gauge that is thought to be nearly identical to the Non-Logo Gauge. Specifically, Model CJ-01 with the description "Electronic Ball Pressure Gauge." Exponent obtained multiple dozens of Exemplar Gauges from both Wilson Sporting Goods (via Paul, Weiss) and other sports equipment retailers

[21] Neither Exponent nor Paul, Weiss was able to procure exemplar gauges identical to the Logo Gauge.

Logo gauges. If the Patriot gauge were a new Model CJ-01 gauge, this would be plausible support for concluding that it was unlikely to be biased high by ~0.3 psi or higher. However, there is no evidence that the Patriot gauge was a new Model CJ-01 gauge or even a new gauge. If Anderson's (older) Logo Gauge had incurred calibration drift upwards since its original purchase, something similar could have occurred with the Patriot gauge. If so, this is surely evidence, however slight, in favor of (rather than against) the proposition that used team gauges might have experienced similar drift.

Exponent's already weak argument becomes vanishingly weak under the "Prioleau" Gauge assumption, which assumes that Anderson used the Non-Logo Gauge for pre-game Colt measurements, thereby rendering moot any argument based on apparent similarity of Colt readings. The issue then becomes, not whether it was "very unlikely that the Logo Gauge would have read similarly to the gauges used **by each team**", but the very different question of whether the Logo Gauge would have read similarly to the gauge **used by the Patriots** (Jastremski's gauge). On this narrower issue, Exponent's argument simply fails to provide evidence to support a statement that it was "very unlikely" that the Patriot gauge could have had similar bias to the Logo Gauge, a gauge, which, after all, was used by a senior NFL official in a championship game.

Other Information

The Timing of Colt Measurements

In one location, the Wells Report stated an astounding uncertainty as to whether Colt measurements took place before Patriot reflation or after:

> there remains some uncertainty about the exact order and timing of the other two events [Colt measurement and Patriot reflation]

That uncertainty could exist about such an elementary order of events indicates that the Wells Report must have received contradictory information about half-time from different NFL officials and that there were no contemporary records to resolve the contradictions.

But, in another location, the Wells Report seemingly placed Colt measurement after Patriot reflation, purporting to explain the NFL's failure to test more than four Colt balls as due to the impending end of the intermission:

> Only four Colts balls were tested because the officials were running out of time before the start of the second half.

However, in Exponent's analysis, they took a third position without supporting argument: that it was "more likely" that Colt measurements took place in mid-intermission (before Patriot reflation) and limited their analysis to this possibility – contrary to Wells' claim that "all permutations" had been analysed.

The opening pressure of Colt balls was ~13.05 psig (the Wells Report considers both 13 and 13.1 psig). At an initialization temperature of 71 deg F, this implies that the Colt balls contained approximately 0.3309 moles of air (based on a football volume of 4237 cm^3). Their half-time pressure using the Non-Logo Gauge was 12.325 psig, equivalent to a temperature of 57.1 deg F, a temperature gain of about 9.1 deg F, 35% recovery to equilibrium of 73.5 deg F.

If the Patriot balls were initialized at 71 deg F with the Logo gauge, this implies that the Patriot balls contained approximately 0.3198 moles of air. Their half-time pressure using the close-to-unbiased Non-Logo Gauge was 11.11 psig, equivalent to a temperature of 51.0 deg F, about one-third of the temperature gain.

If Colt measurements took place towards the end of the intermission, this temperature gain is almost exactly consistent with expectations. If Colt measurements took place around 8 minutes, the temperature gain is about 1.5 deg F less than calculated according to this method, an amount easily accounted for by evaporative cooling.

Intercepted Ball

In addition to the balls measured at half-time, the Wells Report reported measurements on a Patriot ball intercepted shortly before half-time, but Exponent did not analyse this information. The ball was first measured by Colt personnel using a gauge said to be similar to the gauge that the Colts had used for their own pre-game measurement and then turned over to NFL officials, who took the ball to the officials' room and measured it three times using a gauge that NFL officials believed[22] to be Jastremski's gauge (Patriot), yielding values of 11.45, 11.35 and 11.75 psig. The measurement by officials took place between 8:00 p.m. and 8:30 p.m., but no further details were provided in the Wells Report.

Because the average value of the three NFL measurements of the intercepted ball (11.52 psig) is so close to the half-time measurements of Patriot balls using the Logo Gauge (11.49 psig), some commentators have viewed this as support for the Patriot gauge having similar calibration to the Logo gauge. Similarly, the difference between the Colt measurement and the two earliest Patriot measurements (0.4 psi) is almost identical to the bias between the Logo and Non-Logo scales. While these similarities are obviously consistent with the gauge assumptions set out in this article, in my opinion, the time uncertainties do not permit exclusion of other possibilities.

Defective NFL Protocols

The Wells Report also revealed remarkable chaos and inefficiency in the NFL's measurements even under the heightened half-time scrutiny of multiple NFL officials, including NFL Executive Vice President Troy Vincent (who subsequently was responsible for assessing the penalties against Brady and the Patriots):

- At half-time, two NFL officials measured 11 Patriot and 4 Colt balls, each using one of Anderson's two gauges with a third official writing down the measurements. No one recorded which gauge was used for which half-time measurement. The Wells Report concluded that the two officials must have inattentively swapped gauges between

[22] "We believe that [NFL official] Daniel located and used the pressure gauge supplied by the Patriots. We further believe that this is the gauge that John Jastremski considers his normal gauge. It has not been located since the day of the AFC Championship Game. It should be noted that we have not relied upon the air pressure measurements of the intercepted ball in any respect in reaching any conclusions set forth in this Report."

measuring Patriot and Colt balls and that the measurements were reversed for the third Colt ball.

- The NFL did not record observation times of each half-time measurement even though pressures and temperatures were changing through half-time and times are needed for comparison with transients;
- The records were so incomplete and the recollections of officials apparently sufficiently inconsistent that the Wells Report said that there was "uncertainty" even about the order of Colt measurement and Patriot re-inflation during half-time.
- Although NFL officials retrieved all balls at full-time, they did not take measurements of all balls or record measurement times of their measurements. Exponent concluded that this information was too uncertain to be usable. Nor did they take measurements closer to room temperature equilibrium (after two or so hours).
- Although NFL officials were in possession of the relevant Patriot gauge at half-time and its calibration is highly relevant to conclusions of the Wells Report, the NFL lost, misplaced or otherwise released possession of the Patriot gauge and no one could locate it for analysis in the course of the Wells Report.

More professionalism on these protocols would have been expected at a high school science fair – an editorial point that the Wells Report conspicuously failed to make.

Conclusions

The original problem, as framed by Exponent, was whether the observed pressure drop of Patriot balls could be explained by physical or environmental factors, including temperature changes and selection of pregame gauges:

> We then sought to determine whether any combination of the factors listed in 7a through 7d [temperatures at pre-game, on the field and at half-time; timing of half-time measurements; wetness; pre-game gauge use] above (within ranges defined as realistic by Paul, Weiss) suggested pressure levels that matched those recorded on Game Day. If those factors could be set in such a way that the pressures suggested by the transient experiments matched the Game Day measurements, then we could conclude that the Game Day measurements could be explained by physical or environmental factors….

Exponent studied a number of permutations of factors, claiming that none of these combinations accounted for the additional loss of air pressure in Patriot balls or the difference in pressure loss in respect to Colt balls:

> Exponent concluded that, within the range of likely game conditions and circumstances studied, they could identify no set of credible environmental or physical factors that completely accounts for the Patriots halftime measurements or for the additional loss in air pressure exhibited by the Patriots game balls, as compared to the loss in air pressure exhibited by the Colts game balls. Dr. Marlow agreed with this and all of Exponent's conclusions. This absence of a credible scientific explanation for the Patriots halftime measurements tends to support a finding that human intervention may account for the additional loss of pressure exhibited by the Patriots balls.

However, the above analysis refutes their argument by showing that the factors can plausibly be set "in such a way that the pressures suggested by the transient experiments matched the Game Day measurements" as follows:

- Pre-game temperature around 71 deg F
- Logo measurement of Patriot balls and Non-Logo measurement of Colt balls

It is therefore possible to unequivocally say that the "Game Day measurements could be explained by physical or environmental factors", contradicting the key technical finding of the Wells Report. The corollary is that the Wells Report provides no technical basis for concluding that the Patriot balls had even been out of compliance with NFL regulations during the AFC Championship.

The Wells Report also revealed remarkable chaos and inefficiency in NFL protocols and procedures, even in connection with half-time measurements under the additional scrutiny of NFL Executive Vice President Troy Vincent and other senior NFL officials. Had their protocols met reasonable standards, much, if not most, of the present, seemingly false, controversy could have been avoided.

Stephen McIntyre
Climate Audit
June 21, 2015

Appendix 1- Note on Conversion to Master Gauge

For comparison to transients and Exponent converted all measurements in Logo scale and Non-Logo scale to Master Gauge scale using the formulas shown below:

$$Master = \frac{Logo + 0.2836\ psi}{1.050} \quad and\ Master = \frac{Non\text{-}Logo + 0.1444\ psi}{1.015}$$

In multiple half-time (and full-time) paired measurements using the Logo and Non-Logo gauges, a consistent differential of ~0.382 psi between the gauges was observed with a narrow standard deviation of 0.046 psi. Any correct conversion formulas must preserve this differential through successive transformations of Logo to Master and back to Non-Logo scale, but this is not done by Exponent's conversion formulas, which not only result in a lower differential when both are converted to Master Gauge scale, but a differential that is inconsistent. This inconsistency can be directly observed in Exponent's Table 11 which shows a difference of 0.11-0.12 psi between paired measurements converted to Master Gauge scale, when there ought to be no difference.

Table 11. Logo and Non-Logo Game Day averages and their corresponding Master Gauge values (all values are listed in psi).

	Patriots	Converted Master Gauge Pressure	Colts	Converted Master Gauge Pressure
Logo Gauge Average	11.49	11.21	12.74	12.40
Non-Logo Gauge Average	11.11	11.09	12.33	12.29

Nor is this conversion bias uniform across pressures. The figure below shows the Logo-Nonlogo differential after conversion to Master Gauge scale using Exponent's formula, according to various pressures (here using the Non-Logo pressure as an index).

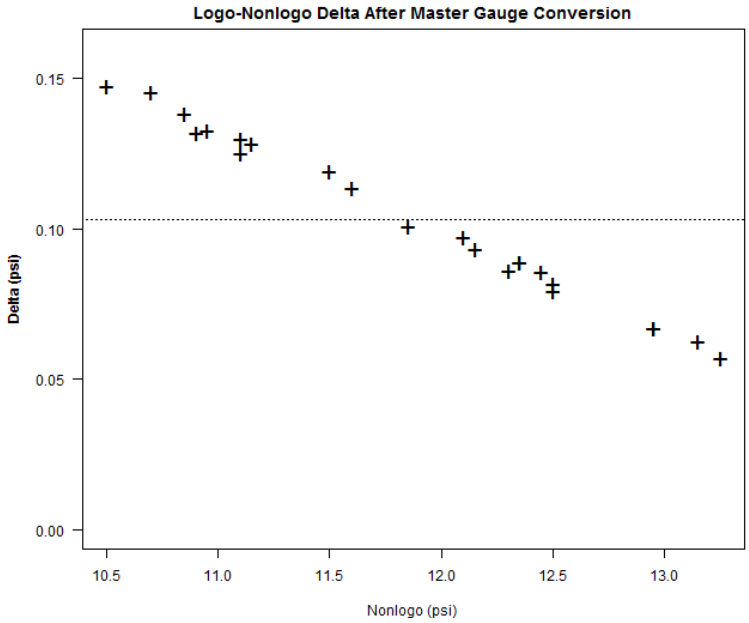

Figure A1. Post-conversion to Master Gauge differential between paired measurements using Logo and Non-Logo scale by Non-Logo Gauge measurement within pair. There is no trend in the difference between actual paired measurements.

Appendix 2 – Newton's Law of Cooling and Negative Exponential Transients

Without evaporative cooling, the solution to the differential equation for Newton's Law of Cooling/Warming is a negative exponential to an asymptote of the following functional form:

$$T(t) = T_{asym} - \left(T_{asym} - T_0\right) * \exp(-C * t)$$

where T_{asym} is the temperature of the officials' room (73-74 deg F), T_0 is the temperature at the end of the first half (48 F) and C is a "restitution coefficient" that is a property of the footballs.

From the temperature transient, the pressure transient can be directly calculated from the Ideal Gas Law:

$$PV = nRT$$

where P is pressure (in pascals), V is the volume of the football (in m3), n is the amount of gas (in moles), R is the gas constant and T is the temperature (in deg K).

Thus, the pressure transients (other than evaporative cooling) are also negative exponentials to an asymptote of the form

$$P(t) = P_{asym} - \left(P_{asym} - P_0\right) * \exp(-C * t)$$

where P_{asym} is the equilibrium pressure at the temperature of the officials' room (73-74 deg F), P_0 is the pressure at the end of the first half at outside temperature (48 F) and C is a "restitution coefficient" that is a property of the footballs. /

All the transients of Wells Report Figures 25, 2/7 and 30 (both dry and wet) are closely fit by negative exponentials with C coefficients close t/o 0.115[23]. On the other hand, the restitution coefficient (from fit to digitized curve) in Wells /Report Figure 16 is noticeably higher (~0.19). The Wells Report does not explain the difference/s. The wet transients in Figure 16 require a biexponential.

[23] The restitution/coefficient in Figure 16 is noticeably higher (~0.19), but no explanation is given in the Wells Report. The wet transients in Figure 16 require a biexponential